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Brief communication

Transient gravity-driven countercurrent two-phase liquid–liquid flow in horizontal and inclined pipes

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1. Introduction

Transient countercurrent two-phase liquid–liquid flow is encountered in a number of engineering applications. In the petroleum industry, for instance, mixtures of crude oil and water are transported to the separation facilities in pipelines consisting of horizontal, uphill, and downhill sections (the hilly terrain pipelines). Scheduled or emergency shutdowns of such pipelines result in the phase separation and countercurrent flow. Free water separated from oil flows downwards and accumulates in the bends joining the downhill sections with the uphill sections of the pipeline. The oil displaced by free water flows upward until the system comes to rest. Transient countercurrent flow of two immiscible liquids also occurs in the pipe-type slug catchers used to store liquids (condensate and free water) produced during pigging of wet-gas pipelines (Price and Thomas, 1998).

Despite its significance, little attention has been paid to this type of multiphase flow. Masliyah and Shook (1978) developed a numerical model for steady laminar countercurrent liquid–liquid flow in an inclined circular tube and obtained an analytical solution for flow between two infinite parallel plates. Zakharov et al. (2000) investigated steady laminar countercurrent vertical flow of two immiscible liquids between parallel plates. To the best of the author's knowledge, no study has been previously performed to investigate transients in stratified countercurrent liquid–liquid flow.

In the studies cited above the so-called rigorous (exact) approach (Wallis, 1982; Taitel, 1994) is used. The local balance equations are solved assuming that the interface position is known a priori. The analytical and numerical solutions based on such an approach predict velocity profiles and shear stress distribution in each phase. Transient two-phase flows in most practical engineering applications have an extremely complex shape of the interfaces even if the flow

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pattern is stratified flow. Predicting the velocity field in each phase that also requires tracking the deformable moving interfaces is a very difficult task (Wallis, 1982; Taitel, 1994) because overwhelming difficulties arise in solving local balance equations (Ishii, 1975). Moreover, short waves (Alkaya et al., 2000) and mixing (Trallero, 1995) at the interface may occur during simultaneous flow of two immiscible liquids in a pipe. The rigorous approach can hardly be applied to such flow patterns. In addition, it should be noted that so detailed description of the flow (2-D or 3-D models) is rarely needed in the practice: an accurate prediction of the average phase velocities, water holdup profiles and the time required to separate the phases is much more important.

In this work, a two-fluid model for transient zero net liquid–liquid flow is presented. A new formulation of the combined momentum balance equation is proposed to account for the water holdup gradient effect. The model predictions are compared with full-scale experimental data for countercurrent oil–water flow. Two mechanisms causing countercurrent liquid–liquid flow in pipes are examined: a non-uniform holdup distribution along the length of the pipeline and the pipe inclination.

2. Problem statement

Two problems are considered in present study. The first problem is schematically shown in Fig. 1a. A horizontal pipeline of diameter D and length L is divided by a valve into two equal sections. Both sections are filled with two immiscible liquids (liquid phase a and b). The density of phase b is greater than that of phase a. The phases are completely separated in both pipeline sections. Initially ($t = 0$), the volume fraction of phase b (in situ holdup) in the first section, α_{b1}, is greater than that in the second section, α_{b2}, and the valve is closed. At $t > 0$, the valve is opened. This study investigates the response of the system on the valve opening.

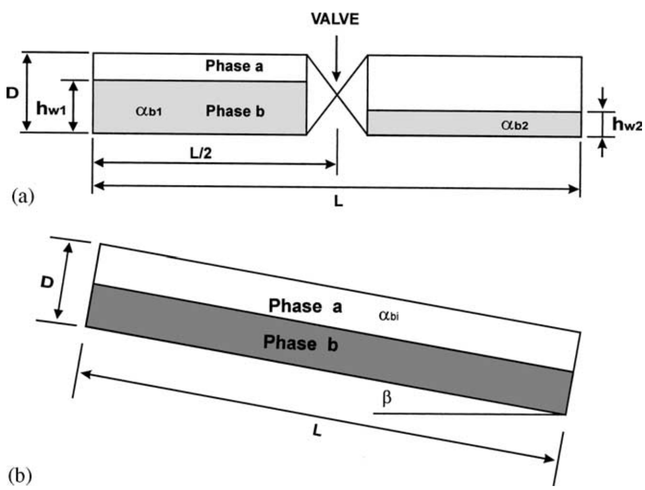

Fig. 1. Countercurrent flow of two immiscible liquids in (a) horizontal pipeline and (b) inclined pipeline.

In the second problem the pipeline is slightly inclined, as shown in Fig. 1b. Two immiscible liquids (a and b) flow simultaneously through the pipeline. Initially, the phases are uniformly distributed along the pipeline length. At $t = 0$, the pipeline is shutdown by closing simultaneously the valves installed at the inlet and the outlet of the line (they are not shown in Fig. 1b). Phase redistribution in the system after shutdown of the pipeline is investigated in this work.

2.1. Governing equations

For an isothermal, incompressible, stratified flow of two immiscible liquids, the basic averaged equations of the two-fluid model can be written as follows:

Mass conservation equation for phase b

$$\frac{\partial}{\partial t}\alpha_b + \frac{\partial}{\partial x}(\alpha_b U_b) = 0, \tag{1}$$

where U_b is the velocity of phase b.

Mass conservation equation for the mixture in an integral form

$$U_a\alpha_a + U_b\alpha_b = U_m, \tag{2}$$

where

$$\alpha_a + \alpha_b = 1, \tag{3}$$

where U_a and α_a are the velocity and the volume fraction of phase a, respectively, U_m is the mixture velocity.

After shutdown of the pipeline the overall flowrate equals zero, thus Eq. (2) reduces to

$$U_a\alpha_a + U_b\alpha_b = 0. \tag{4}$$

Combined momentum equation (Brauner and Maron, 1992)

$$\rho_b\frac{\partial U_b}{\partial t} - \rho_a\frac{\partial U_a}{\partial t} + \rho_b U_b\frac{\partial U_b}{\partial x} - \rho_a U_a\frac{\partial U_a}{\partial x} + (\rho_b - \rho_a)g\cos\beta\frac{\partial h_b}{\partial x} + \frac{\partial(p_{ib} - p_{ia})}{\partial x} = F, \tag{5}$$

where

$$F = -\tau_b\frac{S_b}{\alpha_b A} + \tau_i S_i\left(\frac{1}{\alpha_a A} + \frac{1}{\alpha_b A}\right) + \tau_a\frac{S_a}{\alpha_a A} + (\rho_b - \rho_a)g\sin\beta, \tag{6}$$

where ρ_a and ρ_b are the densities of the phases a and b, respectively; x is the axial coordinate; h_b is the height of the interface; τ_a and τ_b are the wall shear stresses for the phases a and b, respectively; τ_i is the interfacial shear stress; S_a and S_b are the perimeters wetted by the phases a and b, respectively; S_i the perimeter of the interface; g is the gravitational acceleration; p_{ia} and p_{ib} are pressures at the interface at the phases a and b, respectively; and A is the pipe cross-section area.

Phase redistribution in long pipelines ($L \gg D$) is a slow transient process. In slow transients, the gravity and friction forces are dominant, and therefore the inertia effects can be neglected (Taitel and Barnea, 1997). A posteriori estimation of terms in Eq. (5) can also be made to show that this assumption is justified in both cases considered in this study. Thus, the combined momentum equation, Eq. (5), can be written as

$$-\tau_b \frac{S_b}{\alpha_b} + \tau_i S_i \left(\frac{1}{\alpha_a} + \frac{1}{\alpha_b} \right) + \tau_a \frac{S_a}{\alpha_a} + (\rho_b - \rho_a) A g \left(\sin\beta - \cos\beta \frac{\partial h_b}{\partial x} \right) = 0. \tag{7}$$

The examination of Eq. (7) reveals that phase redistribution can be caused either by the pipe inclination ($\beta \neq 0$) or by a non-uniform holdup distribution along the pipeline length ($\partial h_b / \partial x \neq 0$). The direction of phase motion in each particular case is determined by the sum of these two effects.

It is interesting to note that when the system comes to rest ($\tau_a, \tau_b, \tau_i \to 0$) the momentum equation takes the following form:

$$\sin\beta - \cos\beta \frac{\partial h_b}{\partial x} = 0 \tag{8}$$

or

$$\frac{\partial h_b}{\partial x} = \tan\beta. \tag{9}$$

According to Eq. (9) the interface is always parallel to the earth surface when both liquids are at rest. This is an obvious condition for mechanical (static) equilibrium in the system.

2.2. Constitutive relationships

In the present work slow transients in stratified two-phase liquid–liquid flow are investigated, therefore, to evaluate shear stress terms in the momentum equation, the quasi-steady closure relations proposed by Brauner and Maron (1992) were used.

The wall shear stress acting on each phase is expressed in terms of the local velocity of the phase and corresponding friction factor:

$$\tau_k = f_k \frac{\rho_k U_k |U_k|}{2}, \quad k = \text{a}, \text{b}, \tag{10}$$

where f_k is the friction factor for phase k.

The friction factors f_a and f_b in Eq. (10) are evaluated using the adjustable definitions of the equivalent hydraulic diameters (Brauner and Maron, 1992).

The interfacial stress is predicted using the Brauner–Maron correlation for interfacial friction

$$\tau_i = f_i \frac{\rho (U_a - U_b)|U_a - U_b|}{2}, \tag{11}$$

where

$$\begin{aligned} \rho = \rho_a \quad &\text{and} \quad f_i = B f_a \quad \text{for } |U_a| \geqslant |U_b|, \\ \rho = \rho_b \quad &\text{and} \quad f_i = B f_b \quad \text{for } |U_a| < |U_b|. \end{aligned} \tag{12}$$

B in Eq. (12) is the so-called "waviness factor" (Brauner and Maron, 1992) and f_i is the interfacial friction factor. B accounts for the effect of interfacial waviness on the interfacial shear stress. For smooth stratified flow $B \approx 1$, while for wavy stratified or stratified with mixing at the interface flow patterns $B > 1$.

2.3. Initial and boundary conditions

In the first problem, the initial distribution of volume fraction of phase b is uniform in each pipeline section, Fig. 1:

at $t = 0$, for $0 < x < L/2$,

$$\alpha_b(x) = \alpha_{b1}, \tag{13}$$

and at $t = 0$, for $L/2 < x < L$,

$$\alpha_b(x) = \alpha_{b2}. \tag{14}$$

In the second problem, Fig. 2:

at $t = 0$, for $0 < x < L$,

$$\alpha_b(x) = \alpha_{b0}. \tag{15}$$

Both ends of the pipeline are closed during the transients simulated. Thus, the boundary conditions for both cases are:

at $x = 0$ and $x = L$, for $t > 0$,

$$U_b = 0. \tag{16}$$

A detailed description of the numerical method used to solve the governing equations, Eqs. (1), (2) and (7), can be found in a previous article (Fairuzov, 2000). The mesh consisted of 100 cells was used in all the numerical simulation performed.

3. Two-fluid model validation

Eq. (1) is a hyperbolic equation that describes the propagation of phase b holdup disturbances in the pipeline. The studies by Lahey (1992) and by Asheim and Grodal (1998) have shown that void and holdup waves (density waves) in two-phase flow mostly depend on the closure conditions used in two-fluid models. In the present study the empirical correlations for liquid–liquid flow proposed by Brauner and Maron (1992) are used to estimate the wall and interfacial shear stresses. Full-scale experiments were conducted to confirm that these correlations can be used for modeling oil–water flow in large diameter pipelines. A detailed description of the pipeline and equipment used in the experimental study is given in a previous article (Fairuzov et al., 2000).

Table 1 shows a comparison of predicted dimensionless water layer heights with measured values for different operating conditions of the pipeline. A good agreement between the model and the experiment is observed, the predicted water layer height values are within the uncertainty range of the measured values.

Table 1
Comparison of predicted values of water-layer height with measurements

U_m (m/s)	λ_w	h_w/D	
		measured	predicted
0.570	0.338	0.388 ± 0.170	0.389
0.668	0.485	0.388 ± 0.170	0.481
1.271	0.026	0.103 ± 0.034	0.091
1.116	0.030	0.103 ± 0.034	0.100
0.610	0.051	0.103 ± 0.034	0.136
1.411	0.049	0.103 ± 0.034	0.132
0.732	0.263	0.388 ± 0.170	0.335
1.103	0.088	0.178 ± 0.040	0.184
0.524	0.130	0.388 ± 0.170	0.228

4. Countercurrent flow caused by non-uniform phase distribution

In this section, numerical simulations of phase distribution in the pipeline shown in Fig. 1a are presented (problem 1). The length of the pipeline is 500 m and its diameter is 0.3048 m (12 in.). The less dense phase (the phase a) is oil ($\rho_a = 850$ kg/m^3, $\mu_a = 5$ cp) and the more dense phase (the phase b) is water ($\rho_b = 1000$ kg/m^3, $\mu_b = 1$ cp). Initially, the water volume fraction in the first section is 0.4, while in the second section $\alpha_{b2} = 0.1$.

The opening of the valve results in the formation of long density waves ($\lambda \gg D$), which propagate in both pipeline sections (Fig. 2). The water-layer height decreases in the first section and increases in the second one. Phases move in opposite directions and all the flow parameters essentially vary with time (Figs. 3 and 4), transient countercurrent flow occurs. The maximum phase velocities are reached at $t = 0$ s in the valve where $\partial h_b/\partial x \to \infty$ and theoretically $U_a, U_b \to \infty$ (see Eq. (7)). However, in reality, the friction effects limit the flow acceleration in the valve during its opening and the maximum velocities of phases should be finite. The peak values of

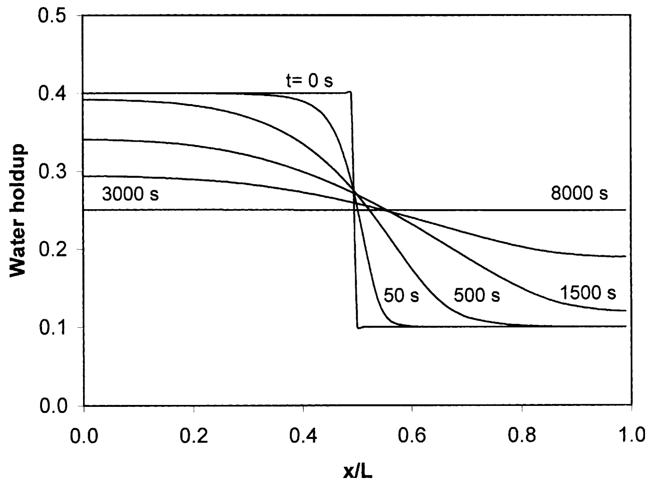

Fig. 2. Phase redistribution caused by a non-uniform water holdup distribution in the horizontal pipeline.

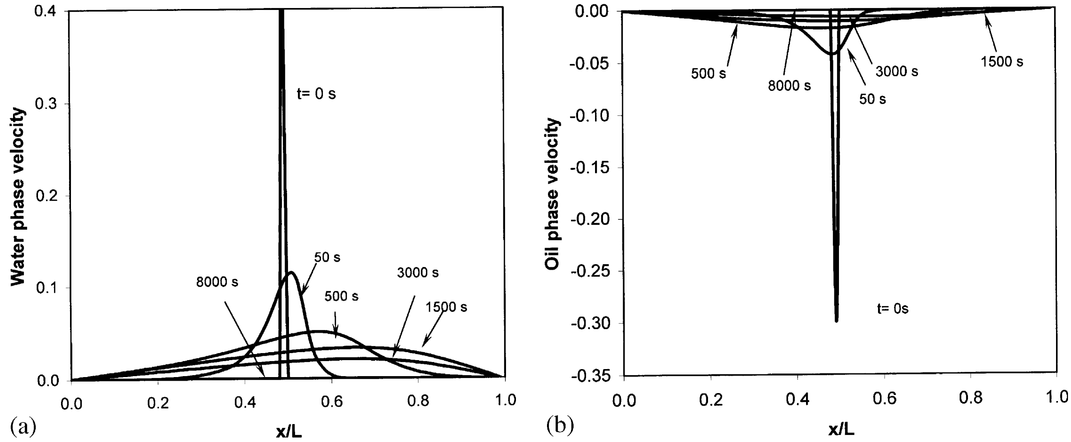

Fig. 3. Transient profiles of phase velocities in the horizontal pipeline.

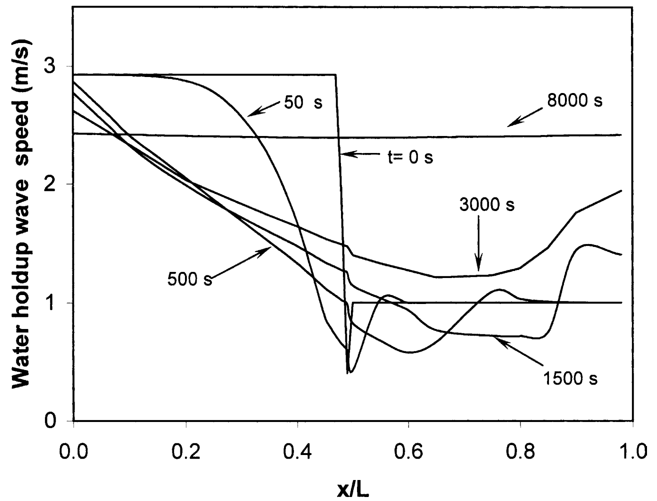

Fig. 4. Holdup wave speeds in the horizontal pipeline.

the phase velocities depend on the design of the valve. A model describing the dynamics of valve opening can be incorporated into the present model to predict the maximum phase velocities at $t = 0$. The valve geometry has an effect on the flow behavior near the valve only during a very short period of time (in the considered case, during first few seconds). Modeling of the valve dynamics is generally of no real practical significance because the time period during which the valve is opened is short as compared to the total time of the fluid transient. In the present study, a final value of the holdup derivative was specified ($\partial h_b/\partial x \approx \Delta h_b/\Delta x$) to evaluate phase velocities in the valve at the initial moment of time.

As can be seen in Figs. 2 and 3 the water holdup perturbations propagate faster in the first section of the pipeline: at $t = 50$ s the phase velocity profiles are not symmetric with respect to the valve location ($X = 1/2L$). This phenomenon may be explained based on a non-linear analysis of the water holdup wave propagation. Eq. (1) may be rewritten in the following form:

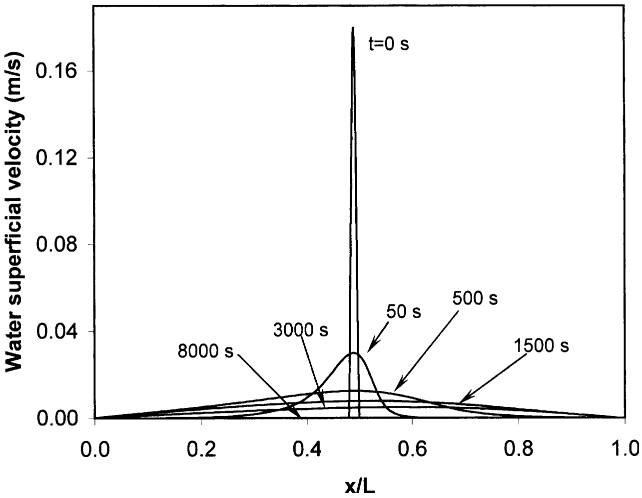

Fig. 5. Transient water superficial velocity profiles in the horizontal pipeline.

$$\frac{\partial}{\partial t}\alpha_b + \left(\frac{\partial U_{bs}}{\partial \alpha_b}\right)_{U_m} \frac{\partial \alpha_b}{\partial x} = 0, \tag{17}$$

where U_{bs} is the superficial velocity of phase b. The term in parentheses in Eq. (17), $c_s = (\partial U_{bs}/\partial \alpha_b)_{U_m}$, is the non-linear holdup wave speed (celerity). The water superficial velocity can be expressed as a function of water holdup, $U_{bs} = f(\alpha_b, U_m)$, with the aid of Eqs. (4) and (7). Thus, to estimate c_s a finite-difference analog for the derivative of U_{bs} with respect α_b can be used. The predicted holdup wave speed profiles are shown in Fig. 4. Initially, $C_{s1} > C_{s2}$ and therefore water holdup disturbances propagate at a higher speed in the first section of the pipeline. It is interesting to note that the local water holdup wave celerities are significantly greater than the local water velocities (Figs. 3 and 4). This indicates that holdup perturbations are traveling along the pipeline much faster than fluid particles in the oil and water. A complex transient behavior of c_s observed in Fig. 4 is attributed to a highly non-linear dependence of the water phase velocity on the water holdup.

After the waves have reached the ends of the pipeline the rate of phase redistribution decreases ($t = 500$ s, Fig. 5). The wave fronts become smooth. As a result, the oil and water flows slow down (Fig. 3). At $t = 8000$ s the interface is flat. This indicates that the system is in an equilibrium state. It is important to note that the inventory (total volume) of each liquid remains constant during the transient. This condition is strictly fulfilled in all numerical simulations presented in this paper.

5. Countercurrent flow caused by pipe inclination

In the second problem considered in the present article (Fig. 1b) the initial water holdup is constant along the whole length of the line, but the pipeline is slightly inclined downward (the inclination angle $\beta = 0.1°$). The initial water holdup is 0.1. All other conditions (pipeline geometry and fluid properties) are the same as in the first example.

The predicted results shown in Figs. 6–8. As the initial water holdup profile is uniform, the phase velocity profiles established immediately after pipeline shutdown are also uniform (Fig. 6, $t = 0$ s). A region of quasi-steady flow ($0.25 < X/L < 0.9$) is observed at $t = 500$ s. As $\partial\alpha_b/\partial X = 0$ in this region, $\partial\alpha_b/\partial t$ equals zero here too according to Eq. (17). At the upper pipeline end the oil phase flowing upward displaces the water phase and a water holdup wave is formed. The holdup wave produced at the upper end moves towards the lower end (Fig. 7). At the lower end the water holdup increases rapidly and the water holdup profile becomes linear soon after the shutdown of the pipeline: a region with flat interface is clearly seen at $t = 500$ s and $0.9 < X/L < 1$. The formation of the region with flat interface may be attributed to a significant increase in the speed of holdup wave caused by water accumulation at the lower end (Fig. 8). In the upper part of the pipeline, the speed of wave propagation decreases and becomes very small (<0.03 m/s) at $t = 3000$ s

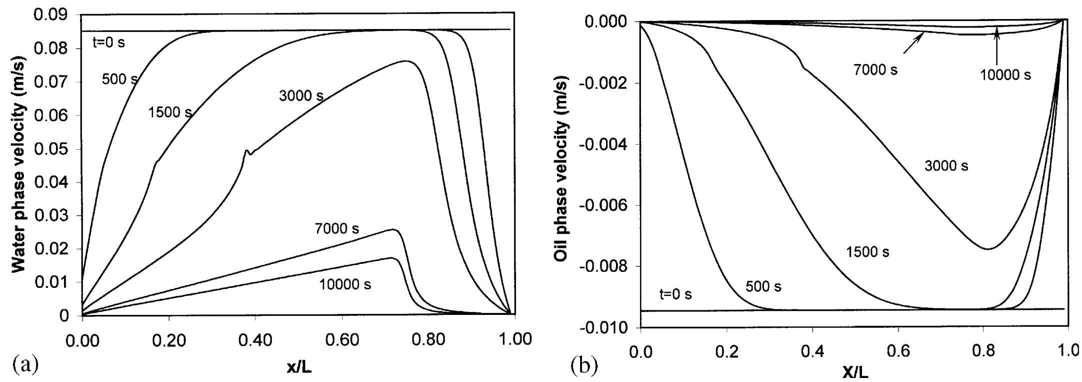

Fig. 6. Transient profiles of phase velocities in the inclined pipeline.

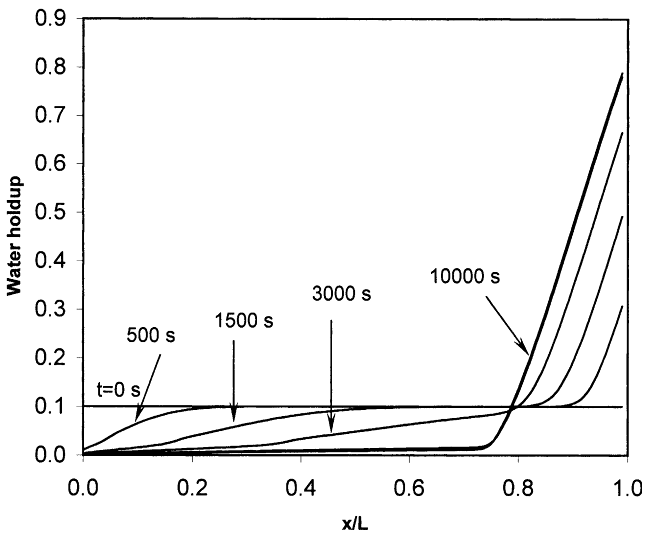

Fig. 7. Phase redistribution in the inclined pipeline.

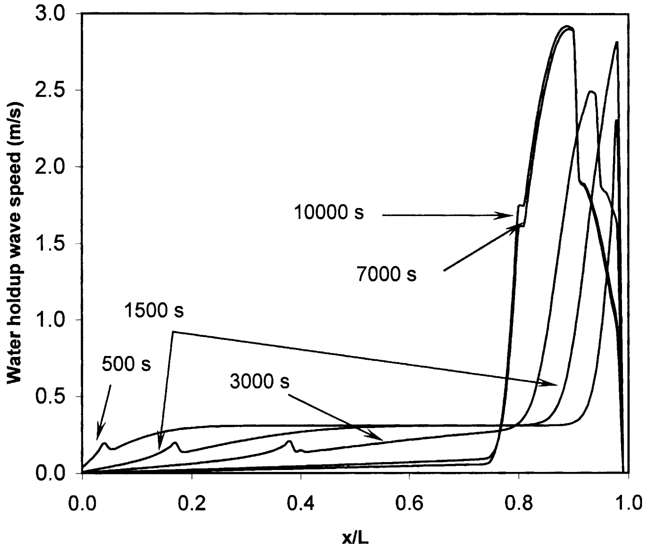

Fig. 8. Holdup wave speeds in the inclined pipeline.

while in the lower part of the pipeline the holdup wave celerities are much larger. Due to this the holdup perturbations generated in the upper part of the pipeline propagate at a significantly higher speed when they reach the pipeline section in which the water accumulation occurs. Thus, the interface at this region is flat (without waves). The shift of the linear holdup profile towards the upper pipeline end, observed in Fig. 7, represents a quasi-steady displacement of the oil phase by the water phase. At $t = 10{,}000$ s the phase redistribution process is almost over. The interface is parallel to the earth surface (Fig. 7). A small amount of water remains in the upper section of the pipeline. This water is flowing downward at a very low velocity (Fig. 6). It takes much time (more than one day in the considered case) to achieve complete phase segregation. In the practice, however, some amount of water always remains in the inclined pipeline sections due to local elastic deformations of the pipe.

It is interesting to note that the holdup wave speed at the lower end of the pipe does not equal zero when $t \to \infty$ (in the considered case, the water holdup and wave speed profiles almost do not change after 10,000 s, Figs. 7 and 8). The water holdup gradient, $\partial \alpha_b / \partial x$, also does not equal to zero at this location (Fig. 7) at $t \to \infty$, because $\beta \neq 0$ (see Eq. (9)). According to Eq. (17) the flow is unsteady at the lower end of the pipe, because $\partial \alpha_b / \partial t \neq 0$ until the system comes to rest. Adding (taking away) water to (from) this part of the pipe will always result in a change of the interface position. The model developed describes correctly this phenomenon.

6. Conclusions

Transient gravity-driven countercurrent flow of two immiscible liquids in horizontal and slightly inclined pipelines was studied numerically in this work. A one-dimensional two-fluid model of the two-phase flow was used to perform numerical simulations of the flow. A new formulation of the combined momentum equation for modeling gravity-driven two-phase flows in horizontal and slightly inclined pipes was proposed. It was shown that such a type of two-phase flow can be produced either by the pipe inclination or by a non-uniform phase distribution along the length of the pipeline. These two mechanisms were investigated separately. It was found that phase redistribution in liquid–liquid systems is controlled by the propagation of holdup waves. The wave propagation speed has a significant effect on the transient water holdup profiles. The final distribution of phases in the pipeline is determined by the pipeline geometry and the inventory of each liquid. This study may contribute towards a better understanding of transient two-phase liquid–liquid flow in pipes.

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